Atmospheric lepton fluxes at very high energy

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Abstract

The observation of astrophysical neutrinos requires a detailed understanding of the atmospheric neutrino background. Since neutrinos are produced in meson decays together with a charged lepton, important constraints on this background can be obtained from the measurement of the atmospheric muon flux. Muons, however, can also be produced as $\mu^+\mu^-$ pairs by purely electromagnetic processes. We use the Z–moment method to study and compare the contributions to the atmospheric muon and neutrino fluxes from different sources (π/K decay, charmed and unflavored hadron decay, and photon conversion into a muon pair). We pay special attention to the contribution from unflavored mesons (η, η', ρ°, ω and ϕ). These mesons are abundant in air showers, their lifetimes are much shorter than those of charged pions or kaons, and they have decay branching ratios of order 10^{-4} into final states containing a muon pair. We show that they may be the dominant source of muons at $E_\mu \gtrsim 10^3$ TeV.

1 Introduction

Atmospheric muons and neutrinos are produced in the showers of high energy cosmic rays in the Earth's atmosphere. Neutrinos can change flavor during their travel from the creation to the detection points, and therefore the observation of their fluxes allows to study their masses, mixings and interactions. Above a minimum energy of a few GeV muons reach the ground, where they can also be observed. Measurements of the atmospheric muon flux provide information about the primary cosmic ray fluxes and about the properties of high-energy hadronic interactions. Moreover, the measurements of the atmospheric muons can be used to obtain precise estimates of the fluxes of ν_μ and $\overline{\nu}_\mu$ *before* flavor oscillation effects. In fact, these measurements have played an important role for the interpretation of the data and the determination of the flavor oscillation parameters. The dominant source of muons and muon neutrinos is the weak decay of charged pions and kaons. These decays always produce $\ell^+\nu_\ell$ or $\ell^-\overline{\nu}_\ell$ pairs, implying a correlation between charged-lepton and neutrino fluxes that can be robustly predicted.

An important goal of present and future experiments is the detection of high-energy neutrinos produced in galactic and extragalactic astrophysical sources [1, 2]. Neutrinos propagate without significant losses (excluding the cosmological redshift) from very distant sources, and one of largest expected signals is an isotropic diffuse flux generated by the ensemble of all extragalactic sources in the universe. For neutrino astronomy, therefore, the atmospheric neutrino fluxes constitute a disturbing background that must be subtracted from the signal. Measurements of the muon flux can help in the determination of this background.

At very high energy it is possible, and indeed virtually certain, that the decay of charged pions and kaons does not remain the dominant source of atmospheric muons and neutrinos. The reason is that these particles are relatively long lived and, because of the Lorentz time dilatation, their decay probability at high energy is strongly suppressed. As an illustration, the decay length of a 10 TeV charged pion is more that 500 Km, around 100 times larger than its interaction length in air. The contribution of particles with a shorter lifetime is therefore likely to become dominant. In particular, the contribution of charmed hadrons is a natural candidate. These particles have large ($\sim$ 10%) branching ratios into semileptonic modes and a lifetime $\tau \sim 10^{-12}$ s, implying a decay probability of order 1 up to energies around 10^7 GeV.

Recently, however, it has been suggested [3] that the dominant source of atmospheric muons of very large energy could be the electromagnetic decay of unflavored mesons into $\mu^+\mu^-$ pairs. These mesons are particles of type $q_f\overline{q}_f$, constituted by a quark and an anti–quark of the same flavor. Neglecting heavy quarks one has 3 scalar (π°, η and η') and three vector (ρ°, ω and ϕ) mesons of this kind. They decay mostly into pions and photons via strong or electromagnetic interactions with a very short lifetime. All of them except the neutral pion (which is below threshold) can decay into a $\mu^+\mu^-$ pair, sometimes together with a photon or a neutral pion ($\eta \to \mu^+\mu^-\gamma$ or $\omega \to \mu^+\mu^-\pi^\circ$), with small branching ratios of order 10^{-4}. These rare decay modes have been neglected in essentially all calculations of atmospheric muons. However, if the multiplicities and the energy spectra of charged pions and unflavored mesons are roughly similar, then they will become the dominant source of atmospheric muons when the average decay probability of charged pions is suppressed by a factor ($\sim 10^{-4}$) of the same order as the branching ratios into the rare modes that contain muons.

In this article we do a critical review of the different contributions to the lepton fluxes at high energies. In Section 3 we estimate the conventional lepton fluxes using the so called Z–moment method, that provides simple analytic expressions. Then we focus on the contribution to the

muon flux from the decay of unflavored mesons. In sections 5 and 6 we evaluate, respectively, the contribution from the prompt decay of charmed hadrons and from γ conversion into a $\mu^+\mu^-$ pair. Finally, we summarize the uncertainties and the implications of our results.

2 Components of the atmospheric lepton fluxes

The atmospheric flux of a lepton type ℓ can be described as the sum of contributions from the decay of different unstable particles:

$$\phi_\ell(E,\theta) = \sum_j \phi_\ell^{(j)}(E,\theta)\,. \tag{1}$$

In this equation E is the energy and θ the zenith angle of the lepton, and the summation runs over all possible parent particles. In this work with *lepton* we refer only to neutrinos and muons. The production of electrons and positrons is dominated by photon conversion in the electromagnetic field of the air nuclei ($\gamma Z \to e^+e^-Z$, with Z the electric charge of the nucleus) and will not be discussed here.

The parent particles that are the source of atmospheric leptons can be naturally divided in three classes. The first class (standard contribution) includes charged pions and kaons that decay via charged-current weak interactions into lepton pairs: ($e^+\nu_e$), ($\mu^+\nu_\mu$) and the charge conjugate states. The observed atmospheric fluxes can at the present time be entirely attributed to this standard contribution.

The second source of atmospheric leptons (charm contribution) is the weak decay of particles that contain a charm (anti)–quark. These decays also generate leptons in ($e\nu_e$) and ($\mu\nu_\mu$) pairs. In addition, $D_s^\pm$ mesons (scalar mesons with a $c\overline{s}$ or $s\overline{c}$ quark content) have a decay branching ratio of $\sim 6.4\%$ into the 2–body mode $\tau^+\nu_\tau$ ($\tau^-\overline{\nu}_\tau$); the subsequent decay of the τ lepton generates a second tau (anti)–neutrino. This chain decay process is the main source of atmospheric ν_τ and $\overline{\nu}_\tau$. The contribution of charmed particles to the atmospheric lepton fluxes is subdominant and currently undetected. It is however expected that this mechanism will overtake the standard contribution at sufficiently high energy.

A third class of parent particles can contribute to the flux of atmospheric muons (but not of neutrinos). This *unflavored contribution* is due to the decay of the unflavored mesons η, η', ρ°, ω and ϕ. These particles have small (order 10^{-4}) branching ratios into final states that include a $\mu^+\mu^-$ pair. The possible significance of this contribution has been discussed in [3], and it will be critically analyzed in the following.

Muon pairs can also be directly produced in Drell–Yan processes and in photon conversions of type $\gamma Z \to \mu^+\mu^- Z$. This last process, despite being suppressed by a factor $(m_e/m_\mu)^2 \sim 2.3\times10^{-5}$ with respect to the production of e^+e^- pairs, is potentially interesting. Its contribution to the muon atmospheric flux will be indicated as $\phi_\mu^{(\gamma)}$ and discussed later.

Taking into account these different sources, the muon and neutrino fluxes can then be expressed as the sum of four and two components, respectively:

$$\phi_{\nu_\alpha}(E,\theta) \;=\; \phi_{\nu_\alpha}^{\rm stand}(E,\theta) + \phi_{\nu_\alpha}^{\rm charm}(E,\theta)\;. \tag{2}$$

$$\phi_\mu(E,\theta) \;=\; \phi_\mu^{\rm stand}(E,\theta) + \phi_\mu^{\rm charm}(E,\theta) + \phi_\mu^{\rm unflav}(E,\theta) + \phi_\mu^{(\gamma)}(E,\theta)\;. \tag{3}$$

3 The standard contribution

Simple analytic expressions for the atmospheric lepton fluxes produced by the decay of charged pions and kaons are described in the textbook [4] by Gaisser (see [5] for additional details). These expressions are obtained under three simplifying assumptions:

(i) The interaction lengths λ_k of all hadrons (labeled by k) are taken constant, neglecting their energy dependence.

(ii) The inclusive spectra of secondary particles j created by the projectile particle k in a hadronic interaction with an air nucleus satisfy the scaling condition:

$$\frac{dn_{kj}}{dE}(E_j;E_k) \equiv \frac{1}{\sigma_k}\,\frac{d\sigma_{kj}}{dE}(E_j;E_k) \simeq \frac{1}{E_k}\;F_{kj}(x)\,, \tag{4}$$

where $x = E_j/E_k$, σ_k is the total inelastic cross section and $F_{kj}(x)$ is the number density of particles j carrying a fraction x of the initial energy after the collision.

(iii) The primary nucleon fluxes are simple power laws of exponent α:

$$\begin{aligned} \phi_p(E_0) =& K\; p_0\; E_0^{-\alpha}\,; \\ \phi_n(E_0) =& K\; n_0\; E_0^{-\alpha} = K\;(1-p_0)\;E_0^{-\alpha}\,. \end{aligned} \tag{5}$$

In the *low-energy limit*, when the parent particle (a charged pion or kaon) decays with probability close to one, the lepton fluxes are isotropic and have a power-law energy spectrum with the same exponent α as the primary nucleon fluxes. The component $\phi_\ell^{(j)}$ takes then the form

$$\phi_\ell^{(j)}(E,\theta) = K\;E^{-\alpha}\,A_j(\alpha)\;Z_{j\ell}(\alpha)\,. \tag{6}$$

$A_j(\alpha)$ is the ratio between the number of nucleons that reach the Earth with energy in the interval $(E,E+dE)$ and the number of particles of type j produced in the same energy interval by primary or secondary particles. This quantity is less than one even if primary nucleons tend to generate many secondary particles of type j, because these particles are produced with lower energy while the ratio is performed at a fixed E. The second quantity in Eq. (6), $Z_{j\ell}(\alpha)$, is analogous, it relates the lepton flux with the flux of its parent particle j. It can be calculated as the $(\alpha-1)$–moment of the inclusive spectrum $F_{j\ell}(x)$ of lepton ℓ from the decay of j:

$$Z_{j\ell}(\alpha) = \int_0^1 dx\; x^{\alpha-1}\;F_{j\ell}(x)\,, \tag{7}$$

where $x = E_\ell/E_j$ and $F_{j\ell}(x)$ is taken in any frame where the parent particle is ultrarelativistic. The quantity $A_j(\alpha)$ includes proton and neutron contributions:

$$A_j(\alpha) = p_0\,A_{pj}(\alpha) + n_0\,A_{nj}(\alpha)\;. \tag{8}$$

It is straightforward (see [4, 5]) to obtain these contributions in terms of Z–factors:

$$A_{pj} \pm A_{nj} = \frac{Z_{pj} \pm Z_{nj}}{1 - Z_{pp} \mp Z_{pn}}\,, \tag{9}$$

where the dependence on α is implicit. Again, the Z–factor $Z_{kj}(\alpha)$ is just the $(\alpha-1)$–moment of $F_{kj}(x)$,

$$Z_{kj}(\alpha) = \int_0^1 dx \; x^{\alpha-1} \; F_{kj}(x) \,. \tag{10}$$

At high energy the decay probability of pions and kaons is suppressed because of the Lorentz time dilatation. When the decay probability of the parent particle is small, using the assumptions (i), (ii) and (iii) it is possible to express the lepton flux from j–decay as

$$\phi_\ell^{(j)}(E,\theta) = K \; E^{-\alpha} \; \frac{\varepsilon_j \, F_{\rm zenith}(\theta)}{E} \; B_j(\alpha) \; Z_{j\ell}(\alpha+1) \,. \tag{11}$$

This energy spectrum is also a power law, but its slope is a unit steeper than in the primary nucleon flux. In addition, it has the strong dependence on the zenith angle described by $F_{\rm zenith}(\theta)$ (shown in Fig. 1). For $\theta \lesssim 60^\circ$ this function is well approximated by a "secant law",

$$F_{\rm zenith}(\theta) \simeq \frac{1}{\cos\theta} \;, \tag{12}$$

whereas for larger zenith angles it keeps growing monotonically, reaching at $\theta \simeq 90^\circ$ a value close to 10. To a good approximation the zenith angle dependence obtained in this *high energy limit* is universal, it does not depend on the parent particle type or the details of the hadronic interactions. The quantity ε_j is the *critical energy* for particle j:

$$\varepsilon_j = \frac{h_0 \, m_j}{c \, \tau_j} \tag{13}$$

that corresponds to the condition where the decay length is equal to the scale height of the air density in the stratosphere $h_0 \simeq 6.36$ Km ($\rho(h) \propto e^{-h/h_0}$). The critical energies for $\pi^\pm$, K_L and $K^\pm$ are approximately 115, 210 and 850 GeV, respectively. After averaging over the creation position, the decay probability of a particle of type j energy E and zenith angle θ, for large energy takes the asymptotic form:

$$P_{\rm dec} = \frac{\varepsilon_j \; F_{\rm zenith}(\theta)}{E} \; \frac{B_j(\alpha)}{A_j(\alpha)} \simeq \frac{\varepsilon_j \; F_{\rm zenith}(\theta)}{E} \tag{14}$$

Note also that the decay Z–factor in Eq. (11) is calculated for the argument $(\alpha+1)$. Finally, $B_j(\alpha)$ is analogous to the quantity $A_j(\alpha)$ defined in the low-energy limit but includes effects due to the distribution of the creation point of particle j. It can be separated as

$$B_j(\alpha) = p_0 \, B_{pj}(\alpha) + n_0 \, B_{nj}(\alpha) \;. \tag{15}$$

Including only the particles produced in nucleon interactions one obtains:

$$B_{pj}(\alpha) \pm B_{nj}(\alpha) = \frac{Z_{pj} \pm Z_{nj}}{1 - Z_{pp} \mp Z_{pn}} \; \left(\frac{\lambda_j}{\lambda_j - \Lambda_N^\pm} \right) \; \ln \left(\frac{\lambda_j}{\Lambda_N^\pm} \right) \;, \tag{16}$$

where the dependence on α is implicit and

$$\Lambda_N^\pm = \frac{\lambda_N}{1 - Z_{pp} \mp Z_{pn}} \;. \tag{17}$$

For pions, the inclusion of *regeneration* effects (the contribution of pions produced in pion interactions) yields the result:

$$B_{p\pi^\pm}(\alpha) \pm B_{n\pi^\pm}(\alpha) = \frac{Z_{pj} \pm Z_{nj}}{1 - Z_{pp} \mp Z_{pn}} \left(\frac{\Lambda_\pi^\pm}{\Lambda_\pi^\pm - \Lambda_N^\pm} \right) \log \left(\frac{\Lambda_\pi^\pm}{\Lambda_N^\pm} \right) , \tag{18}$$

with:

$$\Lambda_\pi^\pm = \frac{\lambda_\pi}{1 - Z_{\pi^+\pi^+} \mp Z_{\pi^+\pi^-}} . \tag{19}$$

Analogous expressions for kaons that include the effect of regeneration are discussed in [5].

Eq. (11) has been obtained under the hypothesis that the decay probability of the parent particle j is small, *i.e.*, $E \gg \varepsilon_j \; F_{\rm zenith}(\theta)$. Correspondingly, the range of validity of the flux in Eq. (6) is $E \ll \varepsilon_j \; F_{\rm zenith}(\theta)$. A useful expression that interpolates between the asymptotic fluxes in (6) and (11) is:

$$\phi_\ell^{(j)}(E,\theta) = \left(K \; E^{-\alpha}\right) \; A_j(\alpha) \; Z_{j\ell}(\alpha) \; \left[1 + \frac{E}{\varepsilon_j \; F_{\rm zenith}(\theta)} \frac{A_j(\alpha)}{B_j(\alpha)} \; \frac{Z_{j\ell}(\alpha)}{Z_{j\ell}(\alpha+1)}\right]^{-1} . \tag{20}$$

The analytic expressions described above have a limited validity, since assumptions (i), (ii) and (iii) are not rigorously correct. The hadronic cross sections grow slowly with energy; the energy scaling (4) is not exact; and the high-energy cosmic ray flux cannot be described as a simple power law because of the steepening at the cosmic ray *knee* ($E_{\rm knee} \simeq 3 \times 10^6$ GeV). In a first approximation, the energy dependence of hadronic interaction lengths and the violation of scaling in inclusive particle distributions can be taken into account just by considering the quantities λ_k and $Z_{kj}(\alpha)$ as (slowly varying) functions of the lepton energy. The calculation of the lepton fluxes for an arbitrary shape of the primary flux is discussed in Subsection 3.2.

3.1 Numerical estimate

The objective in this work is to compare the different contributions to the lepton fluxes at very high energy ($E_\ell \gtrsim 10$ TeV). To estimate the standard contribution from pion and kaon decays one can therefore use Eq. (11). Summing over all parent particles this contribution (for zenith angles $\theta \lesssim 60^\circ$) takes the form:

$$\frac{\phi_\ell^{\rm stand}(E,\theta)}{(K \; E^{-\alpha})} \simeq \frac{\mathbf{E}_\ell(\alpha)}{E \; \cos\theta} , \tag{21}$$

where the constant $\mathbf{E}_\ell(\alpha)$ has dimension of energy and is given by

$$\mathbf{E}_\ell(\alpha) = \sum_{j \in \{\pi^\pm, K^\pm, K_L\}} \varepsilon_j \; B_j(\alpha) \; Z_{j\ell}(\alpha+1) . \tag{22}$$

For a numerical estimate of the quantities $\mathbf{E}_\ell(\alpha)$, we first have obtained the inclusive particle spectra from a MonteCarlo simulation generated by the code Sibyll [6], and then we have deduced the hadronic Z–factors from numerical integrations. The interaction lengths in air have been taken from the PDG fit [7] of hadron–nucleon cross sections, and we have used a Glauber formalism [8] to compute the cross sectios on a nuclear target. The hadronic Z–factors at $E_\ell \simeq 10^6$ GeV that we obtain are shown in Table 1. The characteristic energy for muons,

$\mathbf{E}_\ell$ [GeV]	$\alpha = 2.7$	$\alpha = 3.0$
$(\mu^+ + \mu^-)$	9.3	4.9
$(\nu_\mu + \overline{\nu}_\mu)$	3.4	1.7
$(\nu_e + \overline{\nu}_e)$	0.17	0.10
μ^+	5.2	2.8
μ^-	4.1	2.1
ν_μ	2.2	1.1
$\overline{\nu}_\mu$	1.2	0.6
ν_e	0.10	0.06
$\overline{\nu}_e$	0.07	0.04

Table 1: Characteristic energies $\mathbf{E}_\ell$ (see definition in Eq. (21)) for the lepton fluxes calculated using the Sibyll interaction model. The quantities are evaluated for $E_\ell \simeq 10^6$ GeV.

summing over both charges, is

$$\begin{aligned} \mathbf{E}_\mu(\alpha = 2.7) &\simeq 9.3 \text{ GeV} , \\ \mathbf{E}_\mu(\alpha = 3.0) &\simeq 4.9 \text{ GeV} . \end{aligned} \tag{23}$$

These estimates are given for the value of α relevant below and above the cosmic-ray knee.

The energy dependence of the inclusive particle spectra in the fragmentation region (that dominates the integrand of the Z–factor expression) in the Sibyll code is weak, and the characteristic energies for $E_\ell \simeq 10^6$ GeV (given in Table 1) are approximately 25% lower than the results obtained at $E_\ell \simeq 10^4$ GeV, where the calculation is in reasonable agreement with observations. Because of our poor understanding of hadronic interactions one should attribute around $\sim 30\%$ of systematic uncertainty to the numerical estimates in Table 1 and Eq. (23), but a larger error cannot be excluded.

Finally we note that the absolute prediction for the lepton fluxes also depends on the estimate of the primary nucleon flux [9, 10, 11]. This introduces perhaps the largest uncertainty, because the cosmic-ray composition above the knee is poorly known.

3.2 Lepton yields

To compute the lepton fluxes when the primary spectrum is *not* a power law it is necessary to introduce the lepton yields. The lepton yield $Y_{p(n)\to\ell}(E, E_0, \theta)$ of a primary nucleon of energy E_0 and zenith angle θ gives the average number of leptons ℓ of energy E observable at ground level (where to a good approximation the shower has completed its development) per unit energy:

$$Y_{p(n)\to\ell}(E, E_0, \theta) \equiv \frac{dN_{p(n)\to\ell}}{dE}(E, E_0, \theta) . \tag{24}$$

The lepton flux can then be calculated with a simple integration:

$$\phi_\ell(E, \theta) = \int_E^\infty dE_0 \ \left[\phi_p(E_0)\ Y_{p\to\ell}(E, E_0, \theta) + \phi_n(E_0)\ Y_{n\to\ell}(E, E_0, \theta)\right] . \tag{25}$$

$Y_{p(n)\to\ell}$ can be written as a sum of terms associated to the production and decay of different parent particles:

$$Y_{p(n)\to\ell}(E, E_0, \theta) = \sum_j Y^{(j)}_{p(n)\to\ell}(E, E_0, \theta) , \tag{26}$$

where the index j runs over all species with decay modes containing the lepton ℓ.

Using assumptions (i) and (ii) these yields take simple scaling forms. In the low-energy limit described in the previous section one has

$$\begin{aligned} Y^{(j)}_{N\to\ell}(E,E_0,\theta) &= \frac{1}{E_0}\; Q^{(j)}_{N\mu}(x) = \frac{1}{E_0}\; \left[G_{Nj}\otimes F_{j\ell}\right](x) \\ &= \frac{1}{E_0}\int_0^1 dx_1 \int_0^1 dx_2\; G_{Nj}(x_1)\; F_{j\ell}(x_2)\; \delta(x-x_1\,x_2)\,, \end{aligned} \tag{27}$$

with $x = E/E_0$. $G_{Nj}(x)$ dx is the average number of particles j created in the shower (in interactions of N and secondary particles with air nuclei) with a fraction of energy in the interval $[x, x + \mathrm{d}x]$. The function $G_{Nj}(x)$ is related to $A_{Nj}(\alpha)$ by the expression:

$$A_{Nj}(\alpha) = \int_0^1 dx\; x^{\alpha-1}\; G_{Nj}(x)\,. \tag{28}$$

$G_{Nj}(x)$ can therefore be calculated as the inverse Mellin transform of $A_{Nj}(\alpha)$.

In the high-energy limit the lepton yield takes the form

$$\begin{aligned} Y^{(j)}_{N\to\ell}(E,E_0,\theta) &= \frac{\varepsilon_j}{E_0^2\;\cos\theta}\; R^{(j)}_{N\ell}(x) = \frac{\varepsilon_j}{E_0^2\;\cos\theta}\; \left[\frac{H_{Nj}}{x_1}\otimes F_{j\ell}\right](x) \\ &= \frac{\varepsilon_j}{E_0^2\;\cos\theta}\int_0^1 dx_1 \int_0^1 dx_2\; \frac{H_{Nj}(x_1)}{x_1}\; F_{j\ell}(x_2)\; \delta(x-x_1\,x_2)\,. \end{aligned} \tag{29}$$

Again, $H_{Nj}(x)$ includes the production of hadrons j in primary and secondary interactions inside the shower started by N. This function is related to $B_{Nj}(\alpha)$ by

$$B_{Nj}(\alpha) = \int_0^1 dx\; x^{\alpha-1}\; H_{Nj}(x) \tag{30}$$

and can therefore be calculated as its inverse Mellin transform.

It is straightforward to ckeck that for a power-law primary flux, using Eqs. (27) and (29) to perform the integration in Eq. (25), one recovers for the lepton fluxes the results in Eqs. (6) and (11).

4 Atmospheric muons from unflavored mesons

The decay of the unflavored mesons $\{\eta,\ \eta',\ \rho^\circ,\ \omega,\ \phi\}$ into final states that contain a $\mu^+\mu^-$ pair also contributes to the atmospheric muon fluxes. It is straightforward to use the methods outlined in the previous section to estimate this contribution.

Using the assumptions (i), (ii) and (iii) the muon flux from η mesons is

$$\frac{\phi^{(\eta)}_\mu(E)}{(K\;E^{-\alpha})} \simeq A_\eta(\alpha)\; Z_{\eta\mu}(\alpha) = \left[\frac{Z_{N\eta}}{1-Z_{NN}} + \frac{Z_{N\pi}\;Z_{\pi\eta}}{(1-Z_{NN})\;(1-Z_{\pi\pi})}\right]\; Z_{\eta\mu}(\alpha)\,, \tag{31}$$

with similar expressions for all other unflavored mesons. Eq. (31) provides an estimate of the sum of the μ^+ and μ^- fluxes, taking into account the production of eta mesons both in nucleon and

pion interactions (we neglect the smaller contribution from kaon interactions). The contributions from the decay of unflavored mesons to the positive and negative muon fluxes are identical.

The decay Z–factors are computed from the measured branching fractions into states with muon pairs and from the shape of the muon energy spectra (in Fig. 2). Numerical estimates for these factors are given in Table 2. These estimates have an uncertainty of order 20% associated to the experimental error in the measurement of the relevant branching ratios.

The calculation of the $Z_{N\eta}$ and $Z_{\pi\eta}$ factors obviously requires the modeling of unflavored meson production in nucleon and (less critically) pion interactions. The hadronic Z–factors entering Eq. (31) are

$$\begin{aligned} Z_{N\eta} &= Z_{p\eta} = Z_{n\eta} \ , \\ Z_{\pi\eta} &= Z_{\pi^+\eta} = Z_{\pi^-\eta} \ , \\ Z_{NN} &= Z_{pp} + Z_{pn} = Z_{nn} + Z_{np} \ , \\ Z_{\pi\pi} &= Z_{\pi^+\pi^+} + Z_{\pi^+\pi^-} = Z_{\pi^-\pi^+} + Z_{\pi^-\pi^-} \ , \end{aligned} \tag{32}$$

where we have left the dependence on α implicit and have used isospin symmetry.

The inclusive meson spectra have been obtained from a Sibyll [6] Montecarlo simulation (see Fig. 3), and we have then evaluated the corresponding Z–factors through numerical integration. The results for $\alpha = 2.7$ and $\alpha = 3$ are listed in Table 3. The combination

$$Z_{\rm unflav}(\alpha) = \sum_{j\in\{\eta,\eta',\rho^\circ,\omega,\phi\}} Z_{Nj}(\alpha) \tag{33}$$

is also plotted as a function of α in Fig. 4.

The contribution to the muon flux from unflavored mesons results from the addition

$$\frac{\phi_\mu^{\rm unfl}(E)}{(K\ E^{-\alpha})} = \sum_{j\in\{\eta,\eta',\rho^\circ,\omega,\phi\}} A_j(\alpha)\ Z_{j\ell}(\alpha) = C_\mu^{\rm unflav}(\alpha)\,. \tag{34}$$

From the Z–factors in Tables 2 and 3 we obtain

$$\begin{aligned} C_\mu^{\rm unflav}(\alpha = 2.7) &\simeq 6.2 \times 10^{-6} \ , \\ C_\mu^{\rm unflav}(\alpha = 3.0) &\simeq 3.1 \times 10^{-6} \ . \end{aligned} \tag{35}$$

In the regions of the spectrum where the primary nucleon flux is not well described by a single power law it is possible to compute $\phi_\mu^{\rm unfl}(E)$ from the muon yields. The results of such calculation that we obtain using the Sibyll Montecarlo code are given in Fig. 5. The top line there shows the all-nucleon primary flux. The thick red line is our estimate of the muon flux

x	$Z_{x\mu}(2.7)$ $(\times 10^{-4})$	$Z_{x\mu}(3.0)$ $(\times 10^{-4})$
η	1.37	1.12
η'	0.43	0.35
ρ°	0.33	0.30
ω	1.00	0.86
ϕ	2.15	1.93

Table 2: Decay Z factors for unflavored mesons.

	$Z_{pj}(2)$	$Z_{\pi j}(2)$	$Z_{pj}(2.7)$	$Z_{\pi j}(2.7)$	$Z_{pj}(3.0)$	$Z_{\pi j}(3.0)$
η	0.066	0.094	0.014	0.029	0.0087	0.021
η'	0.052	0.074	0.013	0.027	0.0086	0.020
ρ°	0.054	0.077	0.013	0.026	0.0082	0.019
ω	0.040	0.060	0.010	0.021	0.0066	0.016
ϕ	0.0019	0.0020	0.00038	0.00047	0.00022	0.00029
All	0.22	0.31	0.051	0.10	0.032	0.076

Table 3: Z factors for the production of unflavored mesons in proton and pion collisions with an air nucleus. The inclusive spectra are calculated with the Sibyll Montecarlo code [6]

from the electromagnetic decay of unflavored mesons. We include the conventional $\mu^+ + \mu^-$, $\nu_\mu + \overline{\nu}_\mu$ and $\nu_e + \overline{\nu}_e$ fluxes (from the vertical direction) taking into account only the decay of charged pions and kaons. The data points are from the L3 detector [12].

Inspection of Fig. 5 shows that for the vertical direction ($\theta = 0$) the contribution of unflavored muons overtakes the standard contribution from pion and kaon decay at $E_\mu \simeq 1.6 \times 10^6$ GeV. This result can be also estimated combining Eqs. (23) and (35). The unflavored contribution is isotropic, while the standard contribution grows with increasing zenith angle proportionally to $F_{\rm zenith}(\theta)$.

4.1 Unflavored meson production

We would like to analyze the uncertainties in the modeling of unflavored-meson production in hadronic interactions, and in this subsection we give a qualitative discussion.

The most important quantities for the prediction of the muon flux are the fraction of the projectile energy carried by the unflavored mesons produced in the collision and the shape of their energy spectra. To estimate the energy fraction $\langle x_j \rangle = \langle E_j \rangle / E_0$ carried by the meson type j, one can make some simple considerations. The initial energy E_0 of the projectile is divided among three classes of particles: baryons, antibaryons and mesons,

$$E_0 \simeq E_{qqq} + E_{\overline{qqq}} + E_{q\overline{q}} \,. \tag{36}$$

The energy $E_{q\overline{q}}$ carried by mesons is in turn subdivided among different particle types:

$$E_{q\overline{q}} = \sum_{j \in \{\rm mesons\}} E_j \tag{37}$$

The summation is over the *primary* mesons (before the decay of unstable particles). It is a good approximation to neglect heavy quarks and include in the sum only the 18 mesons that compose the scalar and vector nonets of $SU(3)$. Meson production can be modeled as a two-step process: in the first step $q_i\overline{q}_i$ pairs are created and recombined between each other and with the valence quarks of the interacting nucleons; in the second step the states $q_j\overline{q}_k$ are projected into physical

mesons, whose quark content is known. The scalar unflavored mesons have the quark content

$$
\begin{aligned}
\pi^\circ &= \frac{1}{\sqrt{2}}\,\left(u\overline{u} - d\overline{d}\right)\,; \\
\eta &= \frac{1}{2}\,\left(u\overline{u} + d\overline{d}\right) - \frac{1}{\sqrt{2}}\,s\overline{s}\,; \\
\eta' &= \frac{1}{2}\,\left(u\overline{u} + d\overline{d}\right) + \frac{1}{\sqrt{2}}\,s\overline{s}\,,
\end{aligned}
\tag{38}
$$

whereas for the vector mesons:

$$
\begin{aligned}
\rho^\circ &= \frac{1}{\sqrt{2}}\,\left(u\overline{u} - d\overline{d}\right)\,; \\
\omega &= \frac{1}{\sqrt{2}}\,\left(u\overline{u} + d\overline{d}\right)\,; \\
\phi &= s\overline{s}\,.
\end{aligned}
\tag{39}
$$

Assuming that each light flavor combination $q_j\overline{q}_k$ has on average the same energy and neglecting meson mass differences, the fraction of energy carried by the different species can be calculated in terms of just two parameters: the probability P_s of producing an $s\overline{s}$ pair (with $P_u = P_d = (1 - P_s)/2$), and the probability $P_{\rm scalar}$ to project the state $q_j\overline{q}_k$ into a spin–0 meson. The energy fractions can then be estimated as

$$
\begin{aligned}
&\frac{\langle E_\eta\rangle}{E_{q\overline{q}}} \simeq \frac{\langle E_{\eta'}\rangle}{E_{q\overline{q}}} \simeq P_{\rm scalar}\,\left[\frac{(1-P_s)^2}{8} + \frac{P_s^2}{2}\right] \\
&\frac{\langle E_{\rho^\circ}\rangle}{E_{q\overline{q}}} \simeq \frac{\langle E_\omega\rangle}{E_{q\overline{q}}} \simeq (1 - P_{\rm scalar})\,\frac{(1-P_s)^2}{4} \\
&\frac{\langle E_\phi\rangle}{E_{q\overline{q}}} \simeq (1 - P_{\rm scalar})\,P_s^2\,.
\end{aligned}
\tag{40}
$$

For completeness, the average energy taken by the other mesons is

$$
\begin{aligned}
&\frac{\langle E_{\pi^\circ}\rangle}{E_{q\overline{q}}} \simeq \frac{\langle E_{\pi^+}\rangle}{E_{q\overline{q}}} \simeq \frac{\langle E_{\pi^-}\rangle}{E_{q\overline{q}}} \simeq P_{\rm scalar}\,\frac{(1-P_s)^2}{4} \\
&\frac{\langle E_{K^+}\rangle}{E_{q\overline{q}}} \simeq \frac{\langle E_{K^-}\rangle}{E_{q\overline{q}}} \simeq \frac{\langle E_{K_L}\rangle}{E_{q\overline{q}}} \simeq \frac{\langle E_{K_S}\rangle}{E_{q\overline{q}}} \simeq P_{\rm scalar}\,\frac{(1-P_s)\,P_s}{2}\,.
\end{aligned}
\tag{41}
$$

For the corresponding vector particles (ρ and K^*) one can use Eq. (41) with the substitution $P_{\rm scalar} \to 1 - P_{\rm scalar}$.

For a numerical estimate we can use $E_{q\overline{q}}/E_0 \simeq 0.6$ (with most of the remaining energy carried by one leading baryon) and, following Field and Feynman [13] and the Lund fragmentation algorithm [14], $P_{\rm scalar} \simeq 0.5$ and $P_s \simeq 0.13$. With these assumptions the 5 unflavored mesons carry together an energy fraction $\langle E_{\rm unflav}\rangle/E_0 \simeq 0.18$. This estimate depends only weakly on the values chosen for $P_{\rm scalar}$ and P_s. The minimum value, $\langle E_{\rm unflav}\rangle \simeq 0.13$, is obtained for $P_{\rm scalar} = 1$ and $P_s = 0$.

It is reasonable to expect that the energy spectrum of unflavored mesons is similar or slightly harder than the spectrum observed for pions (which may come from a longer decay chain of unstable primary mesons). A simple 2–parameter form for the inclusive energy spectrum $F_{Nj}(x)$ is:

$$
F_{Nj}(x) = \langle x_j\rangle\,(1 + n_j)\,\frac{(1-x)^{n_j}}{x}\,.
\tag{42}
$$

where $\langle x_j \rangle$ is the energy fraction carried by particle j and n_j a shape parameter in the range 3–4. The Z_{Nj} moments corresponding to (42) are

$$Z_{Nj}(\alpha) = \langle x_j \rangle \; \frac{(1+n_j)\,\Gamma(\alpha-1)\,\Gamma(n_j+2)}{\Gamma(n_j+\alpha)}\,. \tag{43}$$

Note that $Z_{kj}(2) = \langle x_j \rangle$ and $Z_{kj}(3) = \langle x_j \rangle/(n_j+2)$. With these simple considerations it is straightforward to obtain results that are close to those obtained with the Sibyll Montecarlo code.

4.2 Comparison with Pythia

To estimate the systematic uncertainty associated to our calculation we have performed a second calculation of the inclusive spectra of unflavored mesons using the Pythia Montecarlo code. Since Pythia does not support collisions with a nucleus, we have simulated *pp* collisions at 10^6 GeV, and compared the results with the ones from Sibyll for the same type of interactions. The values obtained for the combination $Z_{\rm unflav}(\alpha)$ defined in Eq. (33) are shown in Fig. 4. It is apparent that unflavored meson production in *pp* interactions is qualititively similar in the Pythia and Sibyll codes. Unflavored mesons carry approximately a fraction 0.19 of the projectile energy if produced with the Sibyll code; for the Pythia simulation this energy fraction is reduced to 0.16 (a 20% difference). The energy spectrum in Pythia is however slightly harder. Accordingly, with growing α, the Pythia Z–factor decrease a little more slowly, and the difference between the models is reduced. For $\alpha \simeq 2.6$ the Z–factors for unflavored meson production calculated with the 2 codes coincide. For $\alpha \simeq 3$ the Pythia code gives a result 10% larger. The two codes also show some differences in the relative importance of the different mesons. In Pythia the scalar (vector) mesons are less (more) important with respect to Sibyll.

In summary, the description of unflavored meson production in Sibyll and Pythia in *pp* interactions agrees at the level of 10–15%. This level of agreement is however likely to be an underestimate of the theoretical uncertainties, because the two Montecarlo codes use very similar assumptions.

The calculation of cosmic ray showers requires also the description of hadronic interactions with a nuclear target. The Sibyll code allows to compute the Z factors for both *pp* and *p*–air interactions. The differences are small but not negligible. For a nuclear target the unflavored mesons carry a slightly larger fraction of the energy, but have a softer spectrum (see fig. 4). The first effect is a consequence of the fact that in nuclear interactions the leading baryon is less energetic than in *pp* scattering, and therefore more energy goes into mesons production, on the other hand for a nuclear targer all inclusive spectra are softer.

5 Leptons from charm decay

Weakly decaying charmed particles (D°, D^+, D_s^+, Λ_c and their antiparticles) have a significant probability to decay in semileptonic modes such as $D^\circ \to K^- \mu^+ \nu_\mu$ or $D^\circ \to K^- e^+ \nu_e$, and therefore are sources of atmospheric muons and neutrinos. The production of charmed particles, however, is dynamically suppressed with respect to the production of pions and kaons, and their contribution to the lepton fluxes is usually negligible and remains undetected. On the other hand, charmed particles have a lifetime of order $\tau \sim 10^{-12}$ seconds, and decay with probability close to one up to very high energy. The critical energies ε_j for D°, $D^\pm$, $D_s^\pm$, Λ_c are (0.38,

0.96, 0.85, 2.4)$\times 10^8$ GeV, several orders of magnitude larger than the ones for pions and kaons. Therefore, as the energy grows pion and kaon decay is suppressed and the *prompt* contribution from charm decay will necessarily overtake the standard lepton fluxes.

The estimate of the lepton fluxes from charm decay has been the subject of many studies [15, 16, 17, 18, 19], with results that span a very broad range. In most cases the prediction for this contribution to the muon flux remains always below the one from unflavored meson decay discussed in the previous section. If this were the case the charm contribution would only be observable in measurements of neutrino fluxes. We do not intend to perform here a new calculation of the atmospheric lepton fluxes from charm decay nor a critical review of existing predictions. However, we would like to discuss under which conditions this muon flux is above the expected one from unflavored mesons.

Very likely the production of charmed particles does not obey a scaling law of the type in Eq. (4). A first order estimate of this contribution can however still be obtained using the simple analytic expressions discussed before, treating the hadronic Z–factors as energy-dependent quantities. For $E_\ell < 10^7$ GeV (*i.e.,* a parent charmed particle that decays with probability close to 1) and approximating the primary nucleon flux as a power law, the lepton flux from charm decay can be estimated as

$$\frac{\phi_\ell^{c\bar{c}}(E)}{(K\ E^{-\alpha})} \simeq C_\ell^{c\bar{c}}(\alpha, E) \simeq \sum_{j\in\{D_0,\overline{D}_0,D_s^\pm,\Lambda_c\}} A_j(\alpha, E)\ Z_{j\ell}(\alpha)\,. \tag{44}$$

Including the production of charmed particles in nucleon and pion collisions, the factors $A_j(\alpha, E)$ are:

$$A_j(\alpha, E) = \frac{Z_{Nj}(\alpha, E)}{1 - Z_{NN}(\alpha)} + \frac{Z_{N\pi}(\alpha)\ Z_{\pi j}(\alpha, E)}{[1 - Z_{NN}(\alpha)]\,[1 - Z_{\pi\pi}(\alpha)]}\,, \tag{45}$$

where for simplicity we have assumed equal cross sections for charm production in p/n or $\pi^\pm$ interactions with an air nucleus.

The contribution from charm produced in pion interactions is suppressed by a factor $Z_{N\pi}$, and is expected to introduce just a 20–30% correction. Therefore, for a first order estimate it is sufficient to model charm production in nucleon interactions. To describe the production of the charmed hadron type j in the forward hemisphere we follow the suggestion in [20] and parametrize the inclusive spectrum $F_{Nj}(x, E_0)$ as

$$F_{Nj}(x, E_0) = \frac{A\,\sigma_{c\bar{c}}^{pp}(E_0)}{\sigma_{\rm inel}^{pA}(E_0)}\ p_j\ (n_j + 1)(1 - x)^{n_j}\,, \tag{46}$$

This expression can be integrated in the entire interval $x \in [0, 1]$. The corresponding total charm cross section is

$$\sigma_{c\bar{c}}^{pA}(E_0) \simeq A\ \sigma_{c\bar{c}}^{pp}(E_0)\,, \tag{47}$$

which scales linearly with the mass number A of the target nucleus. The quantity p_j is the fraction of charm events that contain the species j, with $\sum_j p_j = 1$. The Z–factor that corresponds to (46) is

$$Z_{Nj}(\alpha, E) \simeq \frac{A\,\sigma_{c\bar{c}}^{pp}(E)}{\sigma_{pA}^{\rm inel}(E)}\ p_j\ \hat{z}(\alpha, n_j) = \frac{A\,\sigma_{c\bar{c}}^{pp}(E)}{\sigma_{pA}^{\rm inel}(E)}\ p_j\ \frac{\Gamma(\alpha)\,\Gamma(n_j + 2)}{\Gamma(n_j + \alpha + 1)}\,. \tag{48}$$

Note that $\hat{z}(1, n) = 1$, $\hat{z}(2, n) = 1/(n + 2)$, and $\hat{z}(3, n) = 2/[(n + 2)(n + 3)]$.

The modeling of the production of leading charmed baryons remains an important open problem. This process results into a final state with a Λ_c (the longest lived charmed baryon) and, after its decay, high-energy leptons. For this reason we decompose the cross section in 2 parts,

$$\sigma_{c\overline{c}} = \sigma_{\Lambda_c \overline{D}} + \sigma_{D\overline{D}} \ , \tag{49}$$

where the first term accounts for the production of charmed baryons by non perturbative processes. It is natural to expect the energy spectrum of the Λ_c to be significantly harder than the spectrum of D's. Choosing as reference point $n_D \simeq 5$ and $n_{\Lambda_c} \simeq 1$, we obtain

$$\begin{aligned} C_\mu^{c\overline{c}}(\alpha = 3) \simeq & 1.2 \times 10^{-6} \left[\frac{\sigma^{pp}_{D\overline{D}}}{100 \ \mu\text{barn}} \right] \frac{\hat{z}(3, n_D)}{\hat{z}(3,5)} + \\ & + 1.5 \times 10^{-6} \left[\frac{\sigma^{pp}_{\Lambda_c\overline{D}}}{100 \ \mu\text{barn}} \right] \left(0.6 \ \frac{\hat{z}(3, n_{\Lambda_c})}{\hat{z}(3,1)} + 0.4 \ \frac{\hat{z}(3, n_D)}{\hat{z}(3,5)} \right) \end{aligned} \tag{50}$$

This prediction assumes isospin symmetry, that (primary) scalar and vector charmed mesons are produced with the ratio 1/3, and that the production of D_s is suppressed by a factor $\simeq 0.12$ with respect to the production of charmed particles with zero strangeness.

Comparing Eqs. (50) and (35) one can see that the charm contribution to the atmospheric muon flux is below the contribution from unflavored mesons unless the cross section (at $E_0 \simeq 10^6$ GeV) is larger than $\sigma^{pp}_{c\overline{c}} \simeq 100$ μbarn or the energy spectrum of charmed particles is surprisingly hard.

The neutrino spectra generated by charmed particle decay can be estimated including the appropriate energy spectrum of the neutrinos. Because of the $(V - A)$ properties of the matrix element these spectra are a little harder than the corresponding muon spectrum. The decay Z–factors (for $\alpha = 3$) are in the ratios:

$$\begin{aligned} Z_{D\nu_\mu}(3) \simeq Z_{D\nu_e}(3) \simeq 1.25 \ Z_{D\mu}(3) \\ Z_{\Lambda_c\nu_\mu}(3) \simeq Z_{\Lambda_c\nu_e}(3) \simeq 1.16 \ Z_{\Lambda_c\mu}(3) \end{aligned} \tag{51}$$

The different ν/μ ratios for D and Λ_c decay are a consequence of the difference in the available phase space in the two cases. In conclusion one expects that, without the inclusion of neutrino oscillations, the ν_e and ν_μ spectra from charm decay are approximately 20% higher that the corresponding muon flux. This can be considered a robust prediction, in the sense that it is essentially independent of the modelling used for charm production.

The flux of ν_τ generated by the chain decay of $D_s^\pm$ is approximately 30 times smaller than the ν_e or ν_μ fluxes. This estimate is obtained assuming $\sigma_{D_s}/\sigma_D \simeq 0.12$ and taking into account the relevant decay branching ratios and energy spectra.

The decay probability of charmed particles becomes less than unity for energies larger than $\sim 10^7$ GeV, and therefore the expression in Eq. (44) becomes a poor approximation. To describe the resulting lepton fluxes at such energies it is necessary to introduce their critical energies ε_j and use a functional form of the type given in Eq. (20).

6 Photon conversion into $\mu^+\mu^-$ pairs

A potentially interesting source of atmospheric muons is the photon conversion into a pair of muons:

$$\gamma + Z \to \mu^+\mu^- + Z \tag{52}$$

where Z is the electric charge of an air nucleus. This process is suppressed with respect to the production of e^+e^- pairs by a factor of $(m_e/m_\mu)^2 \simeq 2.3 \times 10^{-5}$. However, high energy showers contain a very large number of photons, and it is not immediately obvious that this contribution to the muon flux is entirely negligible.

To estimate the contribution of photon conversions into $\mu^+\mu^-$ pairs to the atmospheric muon flux it is possible to use the same methods discussed above. The development of an electromagnetic shower at high energy is controled by the processes of bremsstrahlung and pair production, which are described by the scaling functions

$$\frac{d\sigma_{e\to e\gamma}}{dv} = X_0\, \frac{A}{N_A}\, \varphi(v)\,, \tag{53}$$

with $v = E_\gamma/E_e$, and

$$\frac{d\sigma_{\gamma\to ee}}{du} = X_0\, \frac{A}{N_A}\, \psi(u)\,, \tag{54}$$

with $u = E_{e^+}/E_\gamma$. The well known expressions for $\varphi(v)$ and $\psi(u)$ can be found, for example, in [22]. In a first approximation the production of muon pairs can be obtained simply rescaling Eq. (54):

$$\frac{d\sigma_{\gamma\to\mu\mu}}{du} \simeq \left(\frac{m_e}{m_\mu}\right)^2 \frac{d\sigma_{\gamma\to ee}}{du}\,. \tag{55}$$

Assuming a power law for the primary nucleon flux and scaling for the hadronic interactions, it is straightforward to obtain that the resulting contribution to the muon flux has the form:

$$\frac{\phi_\mu^{(\gamma)}(E)}{(K\, E^{-\alpha})} \simeq C_\mu^{(\gamma)}(\alpha)\,. \tag{56}$$

The α–dependent constant $C_\mu^{(\gamma)}(\alpha)$ can be calculated as

$$\begin{split} C_\mu^{(\gamma)}(\alpha) \simeq \left[\frac{Z_{N\gamma}}{1-Z_{NN}} + \frac{Z_{p\pi}\, Z_{\pi\gamma}}{(1-Z_{NN})(1-Z_{N\pi})}\right] \times \\ \left[\frac{A(\alpha-1)\,\sigma_\gamma}{A(\alpha-1)\,\sigma_\gamma - B(\alpha-1)\,C(\alpha-1)}\right]\; B(\alpha-1)\; \left(\frac{m_e}{m_\mu}\right)^2 . \end{split} \tag{57}$$

In this expression one can recognize the effects of an hadronic shower that is the source of an electromagnetic one. In Eq. (57) we have left implicit the α dependence of the hadronic Z factors $Z_{N\gamma}$ and $Z_{\pi\gamma}$, the moments of the inclusive photon energy spectra in nucleon and charged pion interactions. The photon spectrum is generated by the decay of neutral pions, with smaller contributions from the decay of η mesons and other hadronic resonances. The quantity σ_γ and the functions $A(s)$, $B(s)$ and $C(s)$ were introduced by Rossi and Greisen [22]:

$$\sigma_\gamma = \int_0^1 du\; \psi(u) \tag{58}$$

and

$$A(s) = \int_0^1 dv\ \varphi(v)\ [1-(1-v)^s] \tag{59}$$

$$B(s) = 2\int_0^1 du\ u^s\ \psi(u) \tag{60}$$

$$C(s) = \int_0^1 dv\ v^s\ \varphi(v) \tag{61}$$

Using in Eq. (57) the numerical hadronic Z-factors obtained with Sibyll at $E \simeq 10^6$ GeV we obtain

$$C_\mu^{(\gamma)}(\alpha = 3.0) \simeq 0.39 \times 10^{-6}\ . \tag{62}$$

For $\alpha \simeq 2.7$ the result is approximately 2.5 times larger.

Comparing this result with Eq. (35) one can see that this contribution to the atmospheric muon flux is approximately one order of magnitude smaller than our estimate from unflavored meson decay. Most predictions of the muon flux from charm decay are also above the result in Eq. (62). Photon conversion into muon pairs is therefore likely to contribute only a small fraction of the atmospheric muon flux even at the highest energies.

7 Summary and Discussion

In this work we have discussed the main contributions to the atmospheric lepton fluxes at very high energy. The standard contribution, due to the decay of charged pions and kaons, is suppressed at high energy because most of these long–lived particles interact, and only a small fraction (inversely proportional to the energy) decays. This suppression results in very steep energy spectra for the lepton fluxes, with a slope approximately one unit larger than the one of the primary cosmic ray flux. The zenith angle distributions of these lepton fluxes have also a characteristic shape, that for θ not too large has the well known $(\cos\theta)^{-1}$ form. Summing over particles and anti–particles (and neglecting neutrino oscillations) the lepton fluxes from pion and kaon decay are in the ratios:

$$\mu \div \nu_\mu \div \nu_e \div \nu_\tau \simeq 1 \div 0.35 \div 0.02 \div 0 \tag{63}$$

These ratios reflect the fact that charged pions generate only $(\mu\nu_\mu)$ pairs, with the muon taking a larger fraction of the parent particle energy. The flux ratios have an uncertainty significantly smaller than the absolute value of the fluxes. The remaining uncertainty is dominated by the error in the estimate of the relative importance of kaon and pion production, and is of order of 15% (20%) for μ/ν_μ (ν_e/ν_μ).

The contributions to the lepton fluxes from charm decay are characterized by a zenith angle distribution that is isotropic for energies below 10^7 GeV, and an energy spectrum that roughly follows the primary nucleon spectrum, with corrections related to the energy dependence of the charm production cross sections. The prediction has however a large uncertainty due to our poor understanding of the dynamics of charmed hadron production. The ratios between the lepton fluxes can be predicted with a much smaller uncertainty

$$\mu \div \nu_\mu \div \nu_e \div \nu_\tau \simeq 1 \div 1.2 \div 1.2 \div 0.04 \tag{64}$$

The decay of unflavored mesons generates an isotropic muon flux, that follows closely the shape of the primary cosmic ray spectrum. Montecarlo codes like Sibyll of Pythia predict that this contribution will overtake the standard contribution (for the vertical direction) at an energy $E \simeq 1.5 \times 10^6$ GeV. Such result reflects that in these Montecarlo codes all mesons that compose the scalar and vector SU(3) nonets are produced with probabilites of the same order (taking into account a suppression for strange quarks). This implies an abundant production of η, η', ρ° and ω mesons, and using the relevant branching ratios one can easily estimate the atmospheric muon flux. The prediction can therefore be considered as quite robust. It should be stressed that this flux is larger than most predictions for the charm contribution, and therefore it may be important for future observations.

We have also considered the flux generated by photon conversion into muon pairs. The muon flux from this source is isotropic and has approximately the same energy spectrum as the flux from the decay of unflavored mesons, but it is approximately 10 times smaller. Photon conversion into muon pairs is therefore likely to be of little phenomenological importance.

It should be stressed that a large uncertainty in the prediction of the lepton fluxes at very high energy arises from our poor knowledge of the primary cosmic ray flux. The relevant energy range is above the cosmic ray knee at $E \simeq 3 \times 10^6$ GeV, where the observations have large errors. Still more important is the fact that the composition of the cosmic ray flux in this energy range is very poorly known. The prediction of the lepton fluxes depends on the so called nucleon spectrum,

$$\phi_0(E_0) = \sum_A A^2 \; \phi_A(E_0 \, A) \, , \tag{65}$$

where $\phi_A(E)$ is the flux for the nuclear species of mass number A and E_0 is the energy per nucleon. In Eq. (65) the contribution of each nuclear species is weighted by a factor A to account for the nucleon multiplicity, and a second factor A is in the jacobian for the transformation from total energy to energy per nucleon. If the cosmic ray flux is a power law of exponent $\alpha \simeq 3$ the nucleon flux scales with composition $\propto \langle A^{2-\alpha} \rangle \simeq \langle A^{-1} \rangle$. A heavy composition corresponds then to a smaller nucleon flux, and to smaller lepton fluxes.

For the normalization of the nucleon flux shown in Fig. 5 the flux of muons from π/K and unflavored mesons decay predicted here is of order

$$\left\langle \Phi_\mu^{\rm stand}(E_{\rm min}) \right\rangle \simeq 400 \;\; \left[\frac{10^6 \text{ GeV}}{E_{\rm min}} \right]^{-3} \;\; ({\rm Km}^2 {\rm yr \, sr})^{-1} \; , \tag{66}$$

$$\Phi_\mu^{\rm unflav}(E_{\rm min}) \simeq 90 \;\; \left[\frac{10^6 \text{ GeV}}{E_{\rm min}} \right]^{-2} \;\; ({\rm Km}^2 {\rm yr \, sr})^{-1} \tag{67}$$

(note the difference in the energy dependence). For the standard contribution we have performed an average over the entire down–going hemisphere. This rough estimates indicate that the contribution to the muon flux from unflavored mesons decay is in principle observable by a neutrino telescope of Km^3.

The most interesting scientific goal of a large neutrino telescopes such as IceCube is the detection of astrophysical neutrinos. The largest signal from astrophysical neutrinos could be the isotropic flux from the ensemble of all extragalactic sources. The signatures of such an astrophysical neutrino signal are: (a) isotropy; (b) a hard energy spectrum; (c) approximately equal fluxes of ν_e, ν_μ and ν_τ. The neutrino fluxes generated by charm decay have also the properties (a) and (b), and equal fluxes for ν_e and ν_μ, and therefore constitute a dangerous

background. Most theoretical models for the production of astrophysical neutrinos predict an energy spectrum harder than what is expected for the charm decay component, however these predictions have important uncertainties

A possible method to separate the astrophysical neutrino signal from the charm decay component is to use measurements of the muon flux to constraint the atmospheric neutrino flux, because charm decay generates approximately equal fluxes of muons, ν_μ and ν_e. The existence of a dominant muon component from unflavored meson decay at very high energy would change critically this type of analysis.

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A Particle Decays

In this appendix we describe the lepton spectra produced in particle decays that are used in this work. The energy spectrum of particle b in the decay of parent particle a can be described in the parent rest frame by the function $G_{ab}(x)$, where $x = 2E_b^*/m_a$ and m_a is the parent particle mass. The spectrum is non vanishing for x in the interval $[x_{\rm min},x_{\rm max}]$. The lower limit is $x_{\rm min} = 2\epsilon$ with $\epsilon = m_b/m_a$, the upper limit is in general $x_{\rm max} \le 2$ and is determined by the masses of the particles in the final state. The normalization of the functions G_{ab} is chosen to be the average multiplicity of particle b in the final state,

$$\langle N_b \rangle = \int_{2\epsilon}^{x_{\rm max}} dx\; G_{ab}(x)\,. \tag{68}$$

In a frame where a is ultrarelativistic the inclusive spectrum of particle b takes the scaling form:

$$\frac{dN_{ab}}{dE_b}(E_a, E_b) = \frac{1}{E_a}\; F_{ab}\,(y)\;, \tag{69}$$

with $y = E_b/E_a$ and

$$F_{ab}(y) = \int_{y+\epsilon^2/y}^{2} dx\; \frac{G_{ab}(x)}{\sqrt{x^2-4\epsilon^2}}\,. \tag{70}$$

Equation (70) assumes that the angular distribution of particle b is isotropic in the parent rest frame. This is true for the decay of spin 0 or unpolarized particles. The decay Z–factor is defined as:

$$Z_{ab}(\alpha) = \int_0^1 dy\; y^{\alpha-1}\; F_{ab}(y)\,. \tag{71}$$

In general, particle b can be present in different decay modes. For example η decays yield muons via the decay modes $\eta \to \mu^+\mu^-\gamma$ and $\eta \to \mu^+\mu^-$. The functions $G_{ab}(x)$, $F_{ab}(y)$ and

$Z_{ab}(\alpha)$ include a sum over all decay channels j where particle b is produced, weighted by the appropriate branching ratios B_j. That is:

$$F_{ab}(y) = \sum_{j \in \{\text{modes}\}} B_j \, F^j_{ab}(y)\,, \quad Z_{ab}(\alpha) = \sum_{j \in \{\text{modes}\}} B_j \, Z^j_{ab}(\alpha)\,, \tag{72}$$

with the spectra for each decay mode normalized to unity.

Two–body decays are very simple to treat. In the rest frame the particle spectrum is monochromatic, and in an ultrarelativistic frame it is flat between appropriate kinematical limits. For the two–body decay of pions and kaons, such as $\pi^+ \to \mu^+ \nu_\mu$, one has [4, 5]:

$$F_{\pi\mu}(y) = \frac{1}{1-\epsilon^2} \; \theta(y - \epsilon^2)\,, \quad F_{\pi\nu}(y) = \frac{1}{1-\epsilon^2} \; \left[1 - \theta(y - 1 + \epsilon^2)\right]\,, \tag{73}$$

with $\epsilon = m_\mu/m_\pi$, and therefore:

$$Z_{\pi\mu}(\alpha) = \frac{1-\epsilon^{2\alpha}}{\alpha(1-\epsilon^2)}\,, \quad Z_{\pi\nu}(\alpha) = \frac{(1-\epsilon^2)^{\alpha-1}}{\alpha}\,. \tag{74}$$

For the decays into a pair of particles with the same mass, such as $\eta \to \mu^+\mu^-$, one has:

$$F^{\mu\mu}_{\eta\mu}(y) = \frac{1}{\sqrt{1-4\epsilon^2}} \; \{\theta[y - y_{\min}(\epsilon)] - \theta[y - y_{\max}(\epsilon)]\}\,, \tag{75}$$

with $\epsilon = m_\mu/m_\eta$ and

$$y_{\min,\,\max}(\epsilon) = \frac{1}{2}\left(1 \mp \sqrt{1-4\epsilon^2}\right)\,. \tag{76}$$

The corresponding Z–factor is:

$$Z^{\mu\mu}_{\eta\mu}(\alpha) = \frac{1}{\sqrt{1-4\epsilon^2}} \; \frac{1}{\alpha} \; [y^\alpha_{\max} - y^\alpha_{\min}]\,. \tag{77}$$

For the decay into three (or more) bodies, the spectrum of the final state particles is determined not only by the particle masses, but also by the matrix element of the decay. As a first approximation one has that the decay of a parent particle into three massless final state particles, just from phase space, has a spectrum (normalized to unity): $G(x) = 2x$, $F(y) = 2(1-y)$ and $Z(\alpha) = 2/(\alpha+\alpha^2)$. To estimate the contribution of unflavored mesons to the muon flux one has to consider 3–body decays such as $\eta \to \mu^+\mu^-\gamma$ and $\omega \to \mu^+\mu^-\pi^\circ$. The spectrum for the latter has been calculated using phase space. For the decay $\eta \to \mu^+\mu^-\gamma$, the spectrum without matrix element is:

$$G^{\mu\mu\gamma}_{\omega\mu}(x) = \frac{1}{C_0(\epsilon)} \frac{(1-x)\sqrt{x^2-4\epsilon^2}}{1+\epsilon^2-x}\,, \tag{78}$$

with $x_{\min} = 2\epsilon$ and $x_{\max} = 1$ and the normalization factor

$$C_0(\epsilon) = \frac{1}{2}\sqrt{1-4\epsilon^2}(1+2\epsilon^2) - \epsilon^2(1-\epsilon^2)\ln\left[\frac{\epsilon^2(1+\sqrt{1-4\epsilon^2})}{1-\sqrt{1-4\epsilon^2}+\epsilon^2(\sqrt{1-4\epsilon^2}-3)}\right]\,. \tag{79}$$

In the ultrarelativistic frame the spectrum becomes

$$F^{\mu\mu\gamma}_{\omega\mu}(y) = \frac{1}{C(\epsilon)}\left\{1 - \frac{\epsilon^2}{y} - y + \epsilon^2 \ln\left[\frac{\epsilon^2 y}{(1-y)(y-\epsilon^2)}\right]\right\}\,, \tag{80}$$

where the kinematical limits $y_{\rm min}$ and $y_{\rm max}$ are again given by expression (76). If the matrix element [23] is included then

$$G_{\omega\mu}^{\mu\mu\gamma}(x) = A(x)\frac{\sqrt{x^2-4\epsilon^2}}{(1-x+\epsilon^2)^2} + B(x)\ln\left[\frac{2\epsilon^2+(1-x)(x+\sqrt{x^2-4\epsilon^2})}{2\epsilon^2+(1-x)(x-\sqrt{x^2-4\epsilon^2})}\right], \tag{81}$$

with

$$A(x) = (1-x)\left[4-11x+7x^2+2(7-8x)\epsilon^2+8\epsilon^4\right], \tag{82}$$

$$B(x) = 2\left[1-4\epsilon^2-2x(1-x)\right]. \tag{83}$$

The normalization $C(\epsilon)$ and the distribution in the ultrarelativistic frame $F_{\omega\mu}^{\mu\mu\gamma}$ must be evaluated numerically (see figure 2).

To describe the decays of charmed particles we have made the simple but reasonably good approximation to treat the dynamics of the charm decay with the matrix element at the the quark level for the decay ($c \to s\ell^+\nu_\ell$) but using the kinematical substitutions: $M(c) \to M(D)$ and $M(s) \to M(K)$ for D decay, and $M(c) \to M(\Lambda_c)$ and $M(s) \to M(\Lambda)$ for Λ_c decay. This allows to compute the decay spectra analytically. Neglecting the muon mass, in the rest frame of the charmed particle the muon and neutrino spectra can be written as:

$$G_{ce(\mu)}(x) = \frac{1}{C(\epsilon)}\frac{12x^2\left(1-x-\epsilon^2\right)^2}{1-x} \tag{84}$$

and

$$G_{c\nu}(x) = \frac{1}{C(\epsilon)}\frac{2x^2\left(1-x-\epsilon^2\right)^2\left[3-(5-2x)\,x+(3-x)\,\epsilon^2\right]}{(1-x)^3}, \tag{85}$$

where $x = 2E^*_{\mu,\nu}/M(c)$, $\epsilon = M(s)/M(c)$ and

$$C(\epsilon) = 1-8\epsilon^2+8\epsilon^6-\epsilon^8-12\epsilon^4\ln\epsilon^2\,. \tag{86}$$

The spectra in a frame where the charmed particle is relativistic can be calculated using (70):

$$F_{ce(\mu)}(y) = \frac{2}{C(\epsilon)}\left\{\left(1-y-\epsilon^2\right)\left[(1-y)\,(1+2y)-(5+4y)\,\epsilon^2-2\,\epsilon^4\right]+6\,\epsilon^4\ln\left(\frac{1-y}{\epsilon^2}\right)\right\} \tag{87}$$

and

$$F_{c\nu}(y) = \frac{1}{3C(\epsilon)}\left\{5-y^2(9-4y)-(27-9y^2)\epsilon^2+\frac{27-9y}{1-y}\epsilon^4-\frac{5-y(4-5y)}{(1-y)^2}\epsilon^6 +6\epsilon^4(3-\epsilon^2)\ln\frac{1-y}{\epsilon^2}\right\}. \tag{88}$$

An important remark is that the energy spectra of the charged leptons is similar, but slightly softer than the corresponding neutrino spectra (fig. 6). From this one can robustly conclude that the expected fluxes of $\mu^\pm$ from charm decay will be approximately 15–20% smaller (depending on the shape of the spectrum and the composition of the parent charm particles) than the corresponding ν_μ and $\overline{\nu}_\mu$ fluxes. The shape of the neutrino and charged lepton spectra in D and Λ_c decay are shown in fig. 6.

The calculation of the spectrum of the $\overline{\nu}_\tau$ produced in the chain decay $D_s^+ \to \nu_\tau + \tau^+ \to \nu_\tau + \overline{\nu}_\tau + X$ should include the effects of the polarization of the τ^+. For the leptonic modes (such as $\tau^+ \to e^+\nu_e\overline{\nu}_\tau$ the problem is identical to the case of the chain decay $\pi \to \mu \to \nu_\mu$. The details for the non leptonic modes will be discussed elsewhere.

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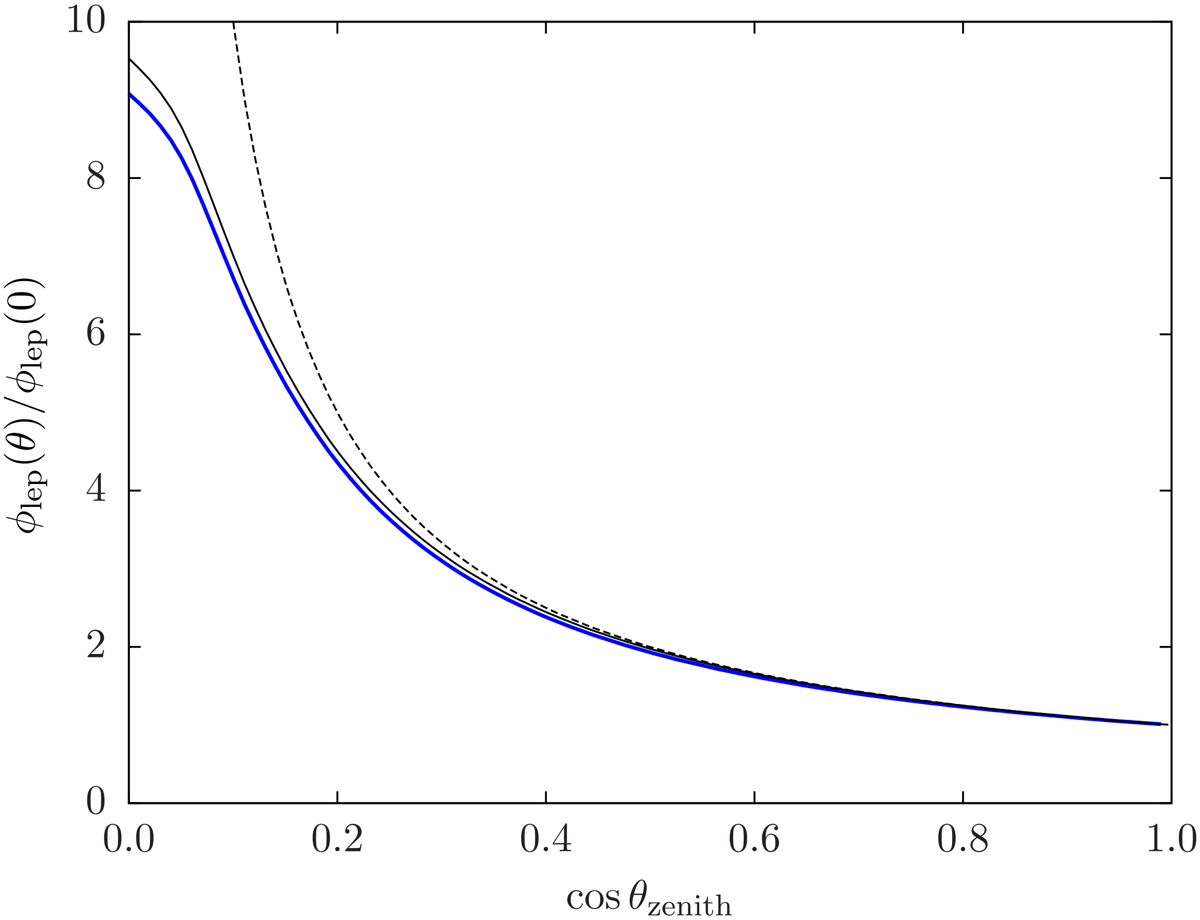

Figure 1: Zenith angle distribution of very high energy lepton fluxes from pion and kaon decay (thick solid line). The dashed line is $(\cos\theta)^{-1}$. The thin solid line represents the curve $[\cos\theta^*(\theta)]^{-1}$, where $\theta^*(\theta)$ is the local zenith angle (the angle with the vertical direction) at the point that corresponds to the column density $t = 200$ (g cm)^2 for the line of sight defined by zenith angle θ at sea level.

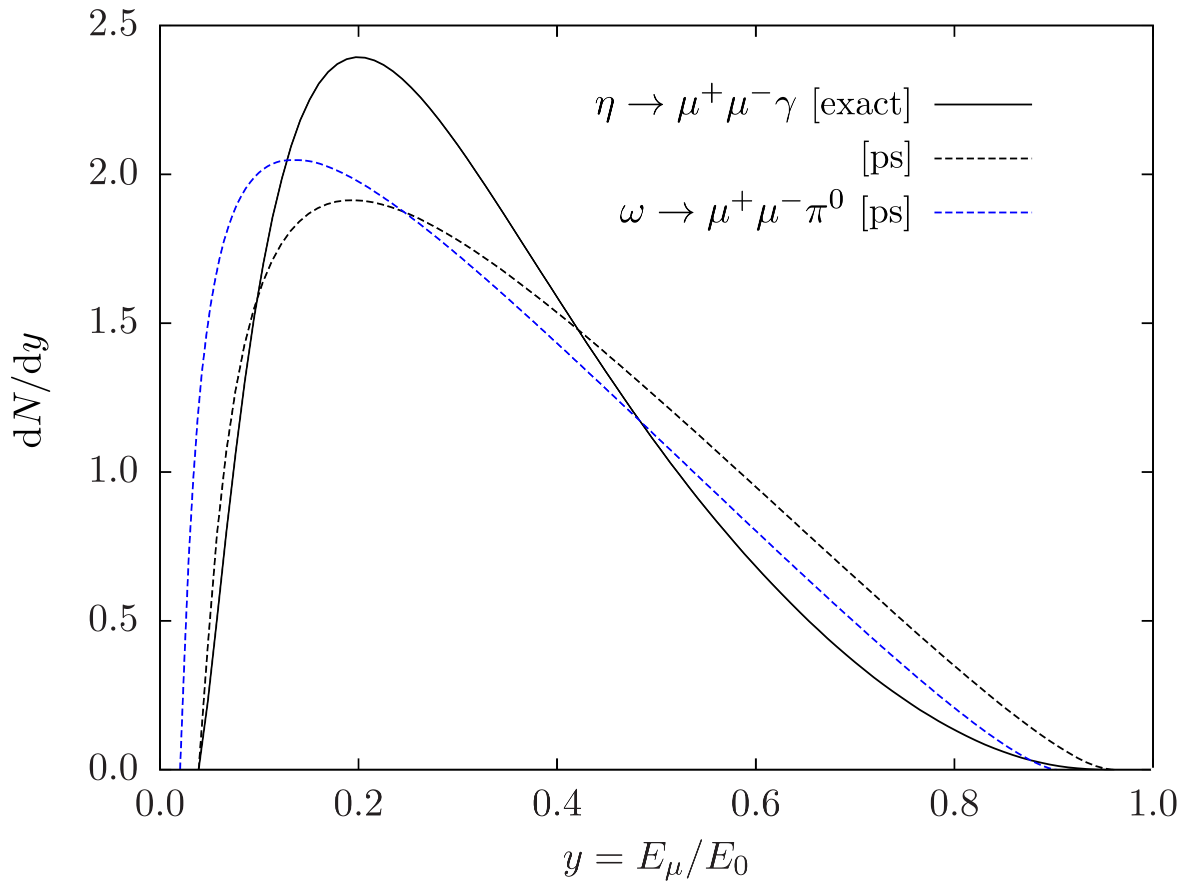

Figure 2: Energy spectra (normalized to unit area) of the muons produced in the 3–body decay of unflavored mesons. The solid lines are for the decay $\eta \to \mu^+\mu^-\gamma$ with (thick) and without (thin) matrix element; the dasheed line is for the decay $\omega \to \mu^+\mu^-\pi^\circ$ (using simple phase space).

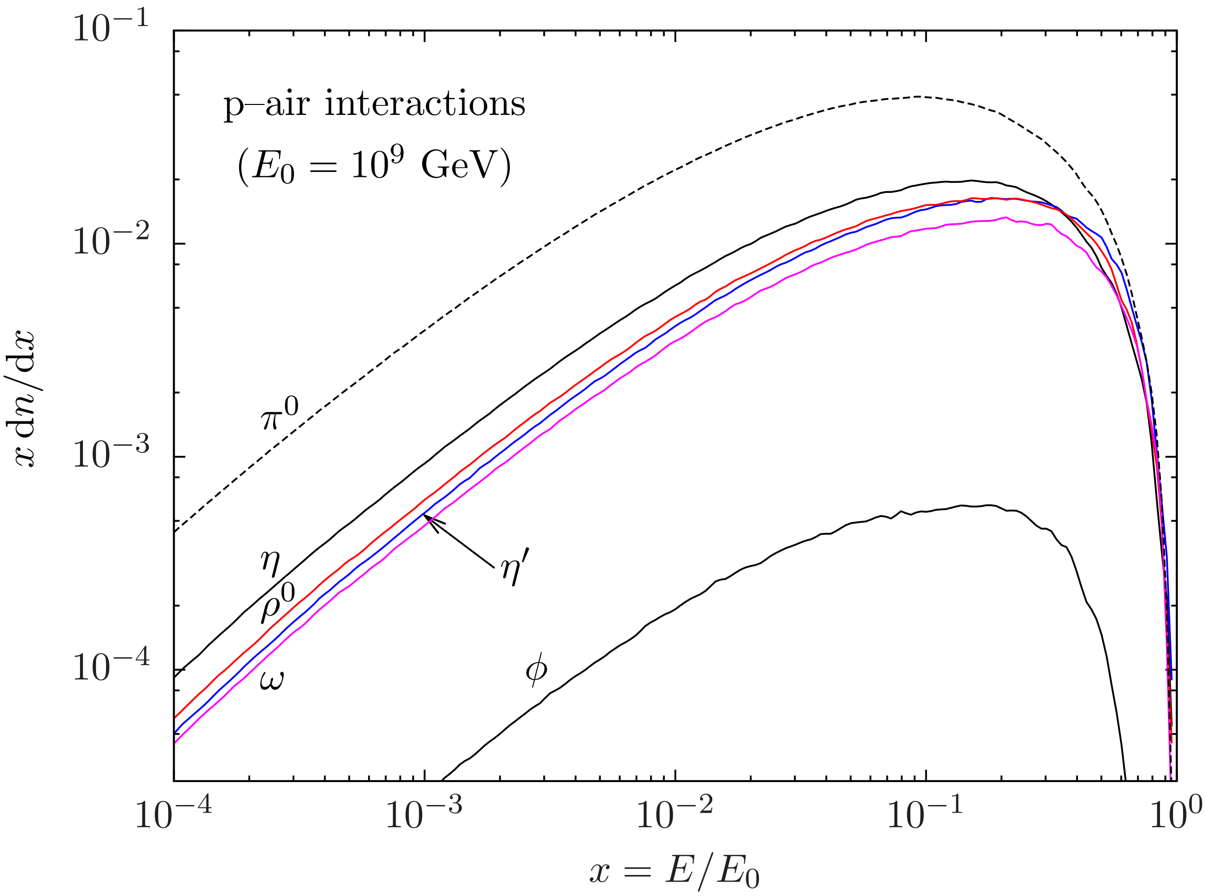

Figure 3: Inclusive spectra of flavorless mesons in p–air interactions (for $E_0 = 10^{15}$ eV). The π° spectrum includes the contribution of the decay of unstable resonances.

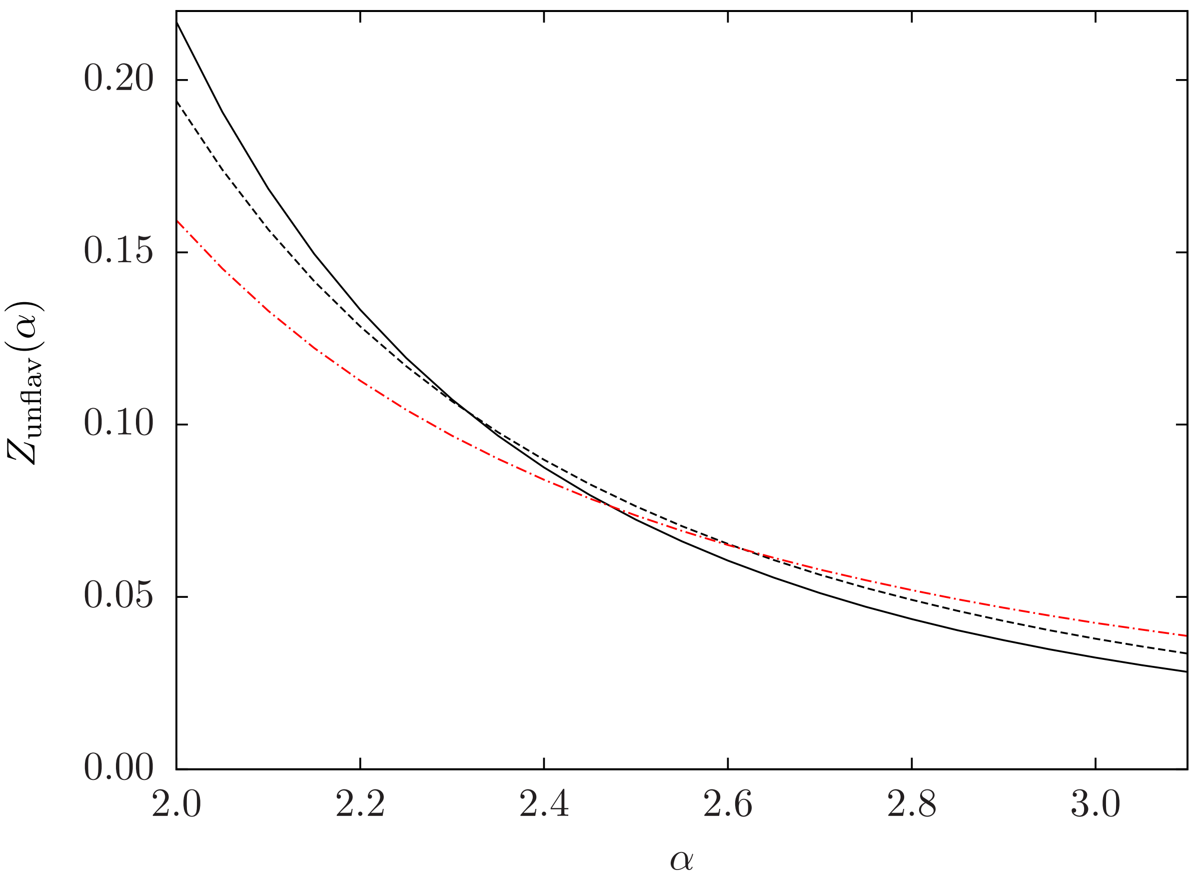

Figure 4: Plot of the quantity $Z_{\rm unflav}(\alpha)$ (defined in equation (33)) as a function of α. The three curves correspond to calculations performed with the Sibyll montecarlo code for pp (dashed line) and p–air interactions (solid line), and with the Pythia code (dot–dashed line) for pp interactions.

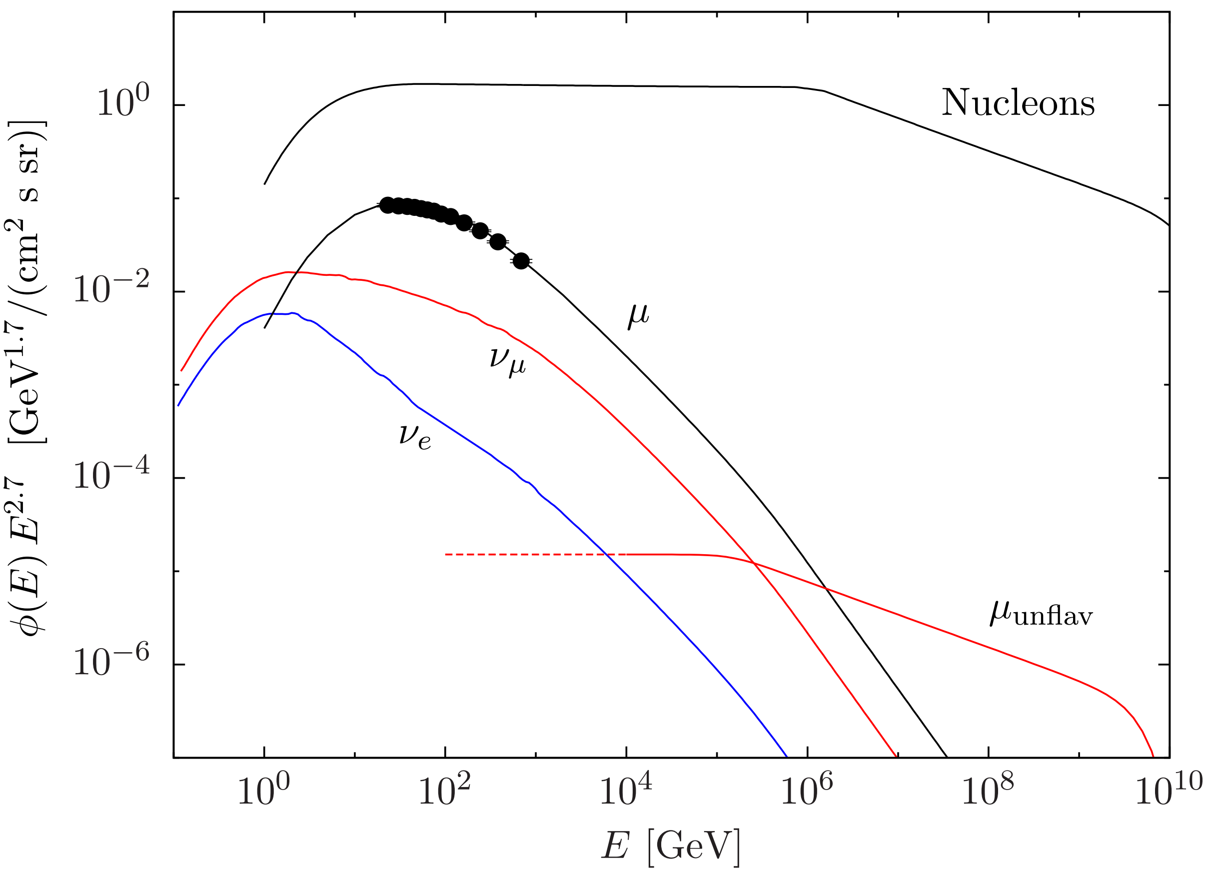

Figure 5: The top line shows the fit of the nucleon flux used in [21]. The curves labeled as μ, ν_μ and ν_e indicate the vertical fluxes of atmospheric $\mu^\pm$, $\nu_\mu + \overline{\nu}_\mu$ and $\nu_e + \overline{\nu}_e$. The calculation of the fluxes extends to higher energy of the results of [21] and coincides with those results for $E \lesssim 30$ TeV. The contribution of the decay of unflavored mesons to the muon flux is shown as the thick (red) curve. The points are the measurement of the muon flux of the L3 detector [12].

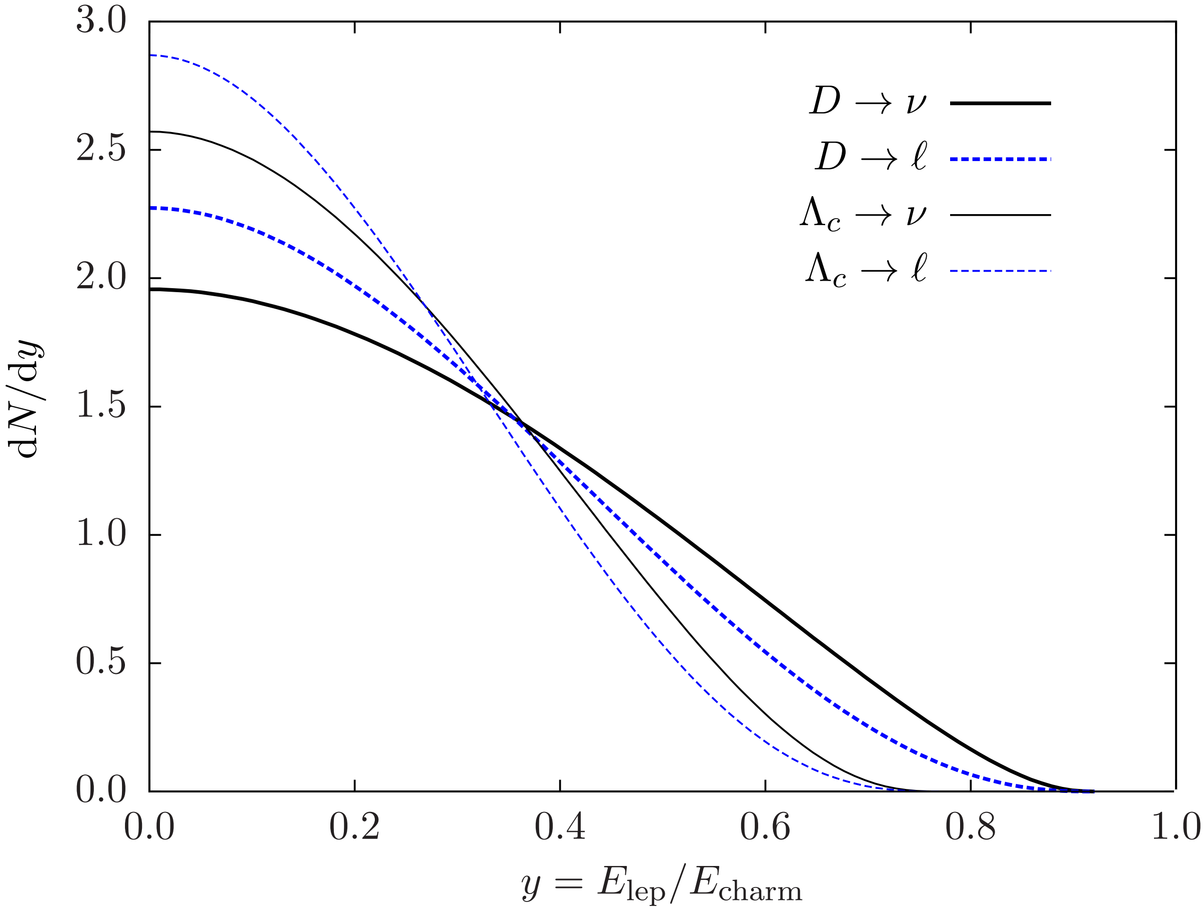

Figure 6: Energy spectra (normalized to unit area) of the muons and neutrinos produced in charmed hadron decays.